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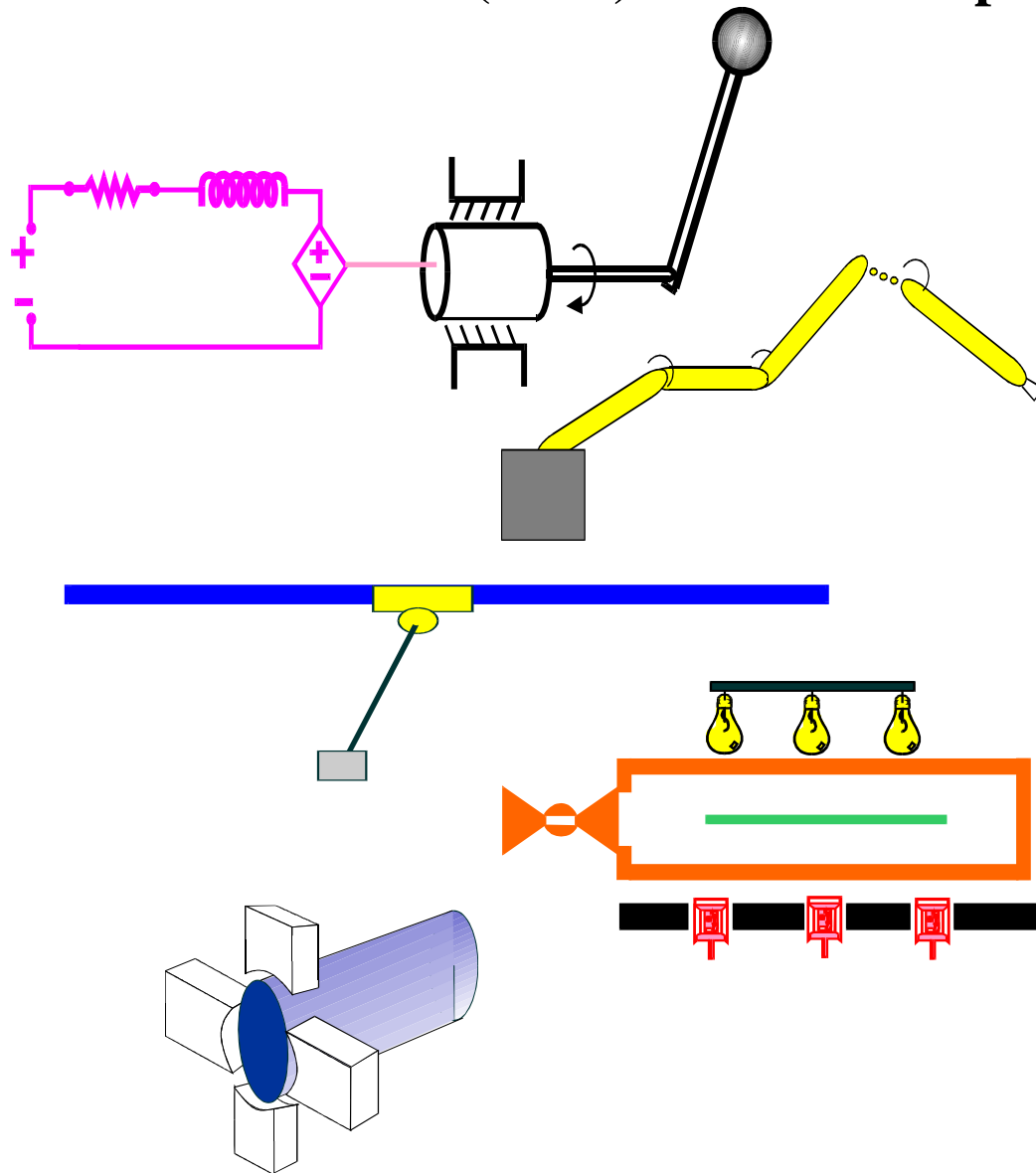# College of Engineering and Science
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Positioning of Large Surface Vessels using Multiple Tugboats

D. Braganza[†], M. Feemster, and D. Dawson

Abstract— In this paper, the positioning of large surface vessels using multiple, autonomous tugboats is investigated. Specifically, the paper investigates the development of two control strategies: i) an adaptive position controller that compensates for select system parameters such as mass and drag coefficients and also for an unknown thrust configuration matrix that is the result of unknown tugboat locations and ii) a robust position control strategy that does not require transmission of information between the tugs. The control design for ii) is facilitated by strategic placement of the tugs about the vessel hull such that the resulting thrust matrix is upper triangular. Performance of both controllers is investigated by simulations for a scaled model of a surface vessel.

I. INTRODUCTION

THE positioning of large surface vessels such as barges, platforms, and ships during low speed maneuvers in such confined environments as harbors is a daunting task. Due to the low speed operation, large vessels that are steered by rudders typically cannot generate sufficient control influence over their position and orientation for delicate docking maneuvers; therefore, multiple tugboats are required to assist in positioning these vessels. In this scenario, tugboats are strategically positioned along the hull of the surface vessel and their efforts are coordinated such that their thrust inputs maneuver the vessel along a desired path. This manual manipulation of the surface vessel with human operators within the process requires precise coordination via radio communication between all involved tugboats. Therefore, its performance is dependent on the skill of the tugboat operators as well as the reliability of the communication network. With the advent of new propulsion technologies such as tunnel thrusters and accurate differential global positioning system (DGPS), operators now posses new tools to aid in this task; however, complete automation coupled with reduced reliance on the communication network between the tugs of this process poses a challenging problem.

In recent years, researchers have been focusing on various marine control problems for surface vessels [1] which can be loosely classified in the following areas: i) position tracking (this includes trajectory tracking and path following also known as maneuvering [2], [3]), ii) way point tracking

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[4], iii) development of autopilots for steering [5], iv) dynamic positioning or station keeping [6], [7], and recently v) formation control [8], [9]. The problem of uncertainty in the dynamic model has also been addressed with the development of adaptive controllers (for example [3], [10]) for systems where the structure of the dynamic model is known and fuzzy/neural network control architectures [5], [11], [12] for systems whose dynamic model structure is not known. Although the positioning problem for fully actuated surface vessels has been considered in [6], [7], these works typically assume that the thrust configuration matrix is completely known. In [13], station keeping experiments were conducted for an over-actuated ship model, and a constrained optimization algorithm was used to allocate the control to the thrusters subject to saturation limits and other constraints. However, this approach also requires the thrust configuration matrix be known *a priori*.

Based on the literature review, it appears that two scenarios within the manipulation of surface vessels with autonomous tugboats may be further matured: i) compensation for an unknown thrust configuration matrix (*i.e.*, the locations of the tugs around the vessel hull are not known), and ii) a control strategy which requires no communication between the tugs. In regards to i), many current surface vessel control strategies do not impose the additional design constraint of an unknown thrust configuration matrix. This omission may be attributed to the fact that previous ship controllers were designed assuming that the vessel under consideration has the ability to generate its own thrust from known actuators positioned at known locations and not from externally connected tugboats. As a result of this unknown thrust matrix, many commonly utilized force allocation (commutation) strategies may be rendered ineffective to distribute the required thrust command among the connected tugs. Therefore, an adaptive, asymptotic position tracking controller that compensates for an unknown thrust configuration matrix as well as such select system parameters as vessel mass and drag coefficients is developed.

In regards to ii), reduced communication between the tugboats holds particular interest in naval warfare applications such as utilizing a swarm of autonomous tugs to recover a ship from a hostile environment. Since each tugboat operates in an independent manner, communication between the tugs is not susceptible to potential electronic jamming which may render other coordinated force allocation strategies ineffective . In order to address this omission of information between the tugs, the design for this case is facilitated by strategic placement of the tugs around the vessel hull such that the thrust configuration matrix is in an upper triangular

form thereby allowing for a hierarchical design technique. In essence, "upper level" tugs the treat the thrusts from the "lower level" tugs as disturbances which are duly compensated for via a neural network feedforward component [14] and a robust control structure.

In addition to the above constraints, the control strategies for both cases are designed in a manner to accommodate manipulation of large vessels where mooring of the tugboats is not tractable (*i.e.* secure attachment of the tugs to the hull cannot be achieved). That is, the control algorithm for each tug is designed such that only a positive or "pushing" thrust is requested (*i.e.*, the tugs cannot pull on the vessel). As a result, a minimum of six tugboats are required for the three degree of freedom positioning problem (x, y positioning and ψ heading control). At this stage it should be mentioned that one major underlying assumption of the control development is the static positioning of the tugboats. That is, it is assumed that once a tugboat has made contact with the hull it remains in contact and does not slip along or change incident angles on the hull. Of course, this assumption may be viewed by some as not practical or too restrictive; however, this assumption allows focus on the adaptive and reduced communication aspects of the control problem. As seen in the subsequent sections, the tugboat dynamics have not been included within the scope of this paper. As a result, future research work will target the inclusion of the dynamics of the tugboats in an effort to remove this assumption.

The paper is structured in the following manner. In Section II, the thrust configuration matrix is studied through a force decomposition analysis and tugboat placement is selected. The 3 DOF kinematic and dynamic model of a vessel influenced by a swarm of tugboats is presented in Section III. In Section IV, an adaptive controller is designed to compensate for an unknown thrust configuration matrix under the assumption of full communication between the tugboats. Subsequently in Section V, a robust positioning controller is developed for the case when communication between the tugboats is not available. Simulation results for both controllers are presented in Section VI to verify their performance. Conclusions and future research efforts are discussed in Section VII.

II. FORCE DECOMPOSITION AND COMMUTATION STRATEGY

The vessel of Figure 1 is assumed to be an unactuated (*i.e.* a barge or disabled ship) whereby the only thrust affecting the vessel is generated from the six tugboats in contact with the vessel's hull. Each tugboat is only allowed to apply a unidirectional thrust input to the vessel (*i.e.*, "push" on the hull). In addition, it is assumed that the position and incident angle of the tugboats are constant (the tugboat dynamics and contact issues such as slipping are not considered within the scope of this paper). The total forces and moments acting on the vessels center of mass (COM) $F = [F_x, F_y, M_z]^T \in \mathbb{R}^3$ is the result of the combined efforts of the six tugboats and are related by the following expression

$$F = B_1 U_1 \tag{1}$$

where $U_1(t) = [u_{1a}, u_{1b}, u_{2a}, u_{2b}, u_{3a}, u_{3b}]^T \in \mathbb{R}^6$ denotes a unidirectional thrust input vector from the six tugboats (*i.e.*, u_i's≥ 0) and $B_1 \in \mathbb{R}^{3 \times 6}$ represents the thrust configuration matrix defined as follows
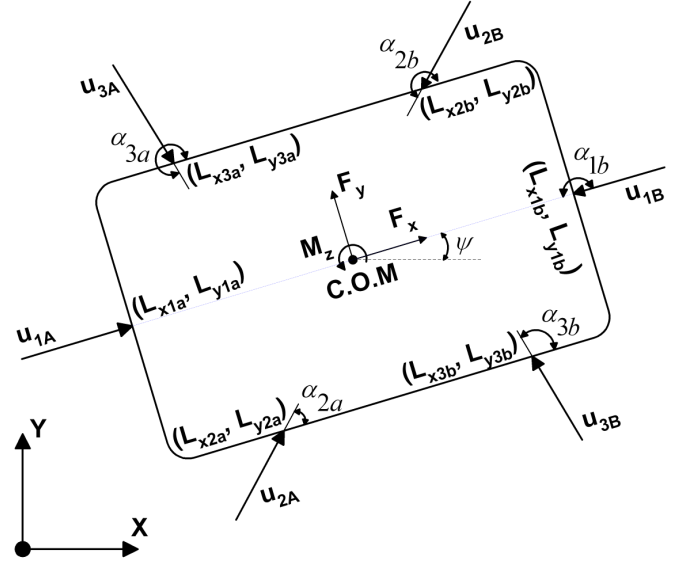

Fig. 1: System description showing the vessel frames.

$$B_1 = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ C_{\alpha 2a} & S_{\alpha 2a} & L_{y2a}C_{\alpha 2a} - L_{x2a}S_{\alpha 2a} \\ C_{\alpha 2b} & S_{\alpha 2b} & L_{y2b}C_{\alpha 2b} - L_{x2b}S_{\alpha 2b} \\ C_{\alpha 3a} & S_{\alpha 3a} & -L_{y3a}C_{\alpha 3a} + L_{x3a}S_{\alpha 3a} \\ C_{\alpha 3b} & S_{\alpha 3b} & -L_{y3b}C_{\alpha 3b} + L_{x3b}S_{\alpha 3b} \end{bmatrix}^T \tag{2}$$

where $C_\theta = cos(\theta)$, and $S_\theta = sin(\theta)$. In order to account for the tugboat's unidirectional influence on the barge, the tugboats are arranged (*i.e.* placed) in an opposing pair fashion about the vessel hull as follows

$$\begin{aligned} \alpha_{2b} = \alpha_{2a} + \pi, \quad L_{x2b} = L_{x2a}, \quad L_{y2b} = L_{y2a} \\ \alpha_{3b} = \alpha_{3a} - \pi, \quad L_{x3b} = L_{x3a}, \quad L_{y3b} = L_{y3a} \end{aligned} \tag{3}$$

which then results in the following force expression

$$F = BU \tag{4}$$

where the $U(t) = [u_1, u_2, u_3]^T \in \mathbb{R}^3$ represents the bidirectional thrust vector, $u_i = u_{ia} - u_{ib}, \forall i = 1, 2, 3$ and the resulting thrust configuration matrix $B \in \mathbb{R}^{3 \times 3}$ is specified as

$$B = \begin{bmatrix} 1 & 0 & 0 \\ C_{\alpha 2a} & S_{\alpha 2a} & L_{y2a}C_{\alpha 2a} - L_{x2a}S_{\alpha 2a} \\ C_{\alpha 3a} & S_{\alpha 3a} & -L_{y3a}C_{\alpha 3a} + L_{x3a}S_{\alpha 3a} \end{bmatrix}^T . \tag{5}$$

Remark 1: Upon closer inspection of the structure of (5), the angle of incidence of tugboat u_{2a} can be selected as follows such that the thrust configuration matrix B is placed in upper triangular form

$$\alpha_{2a} = tan^{-1} (L_{y2a}/L_{x2a}) . \tag{6}$$

Being able to place B into this form allows for a robust control design to be developed that requires no communication between each tug. In essence, "upper level" tug pairs such as u_1 view the thrust inputs from the "lower level" tugs of u_2 and u_3 as disturbances within the x translational dynamics.

Remark 2: It is interesting to note that, the locations of the tugboats specified in (3), can be specified *a priori* such that, no communication is required between the tugboats while maneuvering the vessel. Also, each tugboat will only require its own position and orientation relative to the vessels COM be known.

In the subsequent sections, a bidirectional thrust input $U(t)$ will be designed which promotes the position tracking objective under each case's design constraints. Once a $U(t)$ is defined, the following commutation strategy [15], can be utilized to specify each tug's unidirectional thrust command

$$u_{ia} = \frac{1}{2}\left(u_i + \sqrt{u_i^2 + \gamma_0^2}\right) \tag{7a}$$

$$u_{ib} = \frac{1}{2}\left(-u_i + \sqrt{u_i^2 + \gamma_0^2}\right) \tag{7b}$$

where $i = 1, 2, 3$ and $\gamma_0 \in \mathbb{R}^+$ represents a control gain selected to ensure that the thrust inputs $u_{ij}(t)$ are non-zero, thus preventing the tugboats from losing contact with the vessel.

III. SYSTEM MODELS

The kinematic and dynamic equations for a 3 DOF un-actuated vessel manipulated by six external tugboats can be written as follows [1]

$$\dot{\eta} = R(\psi)\nu \tag{8}$$

$$M(\nu)\dot{\nu} + D(\nu) = BU \tag{9}$$

where $M(\nu) \in \mathbb{R}^{3\times3}$ represents the mass/inertia matrix that includes such effects as added mass, $D(\nu) \in \mathbb{R}^3$ represents the drag, damping, and other parasitic effects, $B \in \mathbb{R}^{3\times3}$ and $U(t) \in \mathbb{R}^3$ denote the thrust configuration matrix and thrust input vector of (4), $\eta(t) = [x(t), y(t), \psi(t)]^T \in \mathbb{R}^3$ represents the composite inertial position (x,y) and heading (ψ) vector of the vessel, $\nu(t) = [u, v, r]^T \in \mathbb{R}^3$ represents the body fixed velocity signals, and $R(\psi) \in SO(3)$ denotes the rotation matrix that translates vessel coordinates into inertial coordinates and is defined as follows

$$R(\psi) = \begin{bmatrix} cos(\psi) & -sin(\psi) & 0 \\ sin(\psi) & cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{10}$$

The subsequent development exploits the following properties of the system model presented in (8)-(9)

Property 1: The mass/inertia matrix $M(\nu)$ and the drag vector $D(\nu)$ are bounded if $\nu(t)$ is bounded.

Property 2: The mass/inertia matrix $M(\nu)$ is positive-definite and symmetric.

Property 3: The rotation matrix $R(\psi)$ satisfies the following properties

$$\begin{aligned} R^T(\psi)R(\psi) = I_3, \qquad \|R(\psi)\| = 1, \forall\psi \\ \dot{R}(\psi) = -\dot{\psi}SR^T(\psi) \end{aligned} \tag{11}$$

where the matrix skew-symmetric matrix $S \in \mathbb{R}^{3\times3}$ is given by

$$S = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{12}$$

In addition to the above system properties, the subsequent control strategies are designed under the following assumptions

Assumption 1: Each tugboat knows its own position $L_i = (L_{xi}, L_{yi})$ and orientation $\alpha_i \forall i = 1a, 1b, .., 3b$, with respect to the vessel center of mass.

Assumption 2: Each tugboat has access to measurements of inertial position $\eta(t)$ and inertial velocity $\dot{\eta}(t)$ of the vessel center of mass. This assumption can be supported in part by Assumption 1. That is, each tugboat is assumed to have knowledge of its position and orientation with respect to the vessels COM. Since each tugboat also has access to measurements of its inertial position (typically available through DGPS), then using the geometric relationship of its hull location relative to the vessels center of mass, each tugboat can formulate the inertial position and orientation of the vessel.

Assumption 3: The tugboats remain in contact with the vessel's hull at all times. They also remain at fixed locations with respect to vessel's center of mass, and do not change incident angles with respect to the hull thereby resulting in a constant thrust configuration matrix B.

IV. CASE I: ADAPTIVE CONTROL DESIGN

In this section, an adaptive position control strategy is developed for the vessel dynamics of (9) with the main objective of promoting vessel tracking along a desired path $\eta_d(t) \in \mathbb{R}^3$ (it is assumed that the selected vessel trajectory is sufficiently smooth in the sense that $\eta_d(t)$, $\dot{\eta}_d(t)$, $\ddot{\eta}_d(t)$, $\dddot{\eta}_d(t) \in \mathcal{L}_\infty$). In addition, the design must compensate for the fact that the tugboats are dispersed at unknown locations around the hull. As a result, the thrust configuration matrix of (5) is partially unknown (one must remember that each tug is assumed to know its location with respect to the vessel's center of mass). In addition, the control design will also compensate for the coefficients associated with the mass matrix $M(\nu)$ and the parasitic effects encapsulated in $D(\nu)$. As a result of the uncertainty in B, the coordinated control input $U(t)$ consists of coupling terms of all the tugs; therefore, communication between the tugboats is required (*i.e.*, this implies the calculation of the individual tugboat thrust inputs $u_i(t)$ will be in some part dependent on information from other tugboats).

A. Transformed System Model

In order to ease the development of the adaptive controller, the system model presented in (8)-(9) is rewritten in the following form [1]

$$M^*(\eta)\ddot{\eta} + D^*(\nu,\eta,\dot{\eta}) = JU \tag{13}$$

where the transformations $\nu = R^T(\psi)\dot{\eta}$ and $\dot{\nu} = R^T(\psi)\ddot{\eta} - \dot{\psi}SR^T(\psi)\dot{\eta}$ were utilized, The transformed system matrices

$M^*(\eta) \in \mathbb{R}^{3\times3}, D^*(\nu, \eta, \dot{\eta}) \in \mathbb{R}^3, J(\eta) \in \mathbb{R}^{3\times3}$ in (13) are defined in the following manner

$$M^* = R(\psi)M(\nu)R^T(\psi)$$
$$D^* = R(\psi)D(\nu) - \dot{\psi}R(\psi)M(\nu)SR^T(\psi)\dot{\eta} \quad (14)$$
$$J = R^T(\psi)B.$$

It should be noted here that the positive definiteness and symmetric attributes of the original mass/inertia matrix $M(\nu)$ are preserved with $M^*(\eta)$ in (14) (*i.e.*, $M^*(\eta)$ is positive definite and symmetric). In addition, it is assumed that parametric uncertainty associated with the system matrices $M^*(\eta)$ and $D^*(\nu, \eta, \dot{\eta})$ is linearly parameterizable. Also, it is assumed that the control input term $J(\eta, \theta_1)U$ can be manipulated into the following form

$$JU = Y_1\theta_1 \,\forall U \in \mathbb{R}^3 \quad (15)$$

where $Y_1(U, \eta) \in \mathbb{R}^{3\times p_1}$ represents a known, measurable regression vector and $\theta_1 \in \mathbb{R}^{p_1}$ represents a vector of unknown, positive constants. In order to prevent potential control singularities, it is assumed that the upper and lower bounds for the parameter θ_1 are known such that $J(\eta, \theta_1)$ is invertible. The bounds for each element in θ_1 can be calculated as follows

$$\underline{\theta}_{1i} \leq \theta_{1i} \leq \overline{\theta}_{1i} \quad (16)$$

where $\underline{\theta}_{1i}, \overline{\theta}_{1i} \in \mathbb{R}$ denote the known lower and upper bounds, respectively, for the ith element of θ_1.

B. Tracking Error System Development

To quantify the positioning objective, a position tracking error signal $e(t) \in \mathbb{R}^3$ is defined as follows

$$e \triangleq \eta - \eta_d \quad (17)$$

where the desired trajectory $\eta_d(t)$ and its first two derivatives $\eta_d(t), \dot{\eta}_d(t), \ddot{\eta}_d(t) \in \mathbb{R}^3$ are known by all the tugboats and are bounded functions of time.

Remark 3: Since $\eta_d(t)$ is explicitly dependent on time, the tugboat group must be synchronized in some fashion. The most simplistic method is to reset clock counters on all the tugs upon deployment; however, one would need to know when all tugs have established contact in order to initiate the maneuvering control strategy. Current research efforts are underway that focus on the approach problem of the autonomous tugboats. Since some tugs may be required to circumnavigate around to the far side of the ship, contact time may vary among group members; therefore, one portion of the focus of this research is the estimation of contact time of all tugs from known initial coordinates. One may consider broadcasting a global initiate command; however, it is desired to keep wireless/radio communication at a minimum so as to reduce susceptibility to electronic interference.

In order to ease the design and analysis of the second order system, the filtered tracking error signal $r(t) \in \mathbb{R}^3$ is defined as follows

$$r \triangleq \dot{e} + \alpha e \quad (18)$$

where $\alpha \in \mathbb{R}^+$ represents a constant control gain. After taking the time derivative of $r(t)$, multiplying through by the mass matrix $M^*(\eta)$, and substituting in the system dynamics of (13), the open-loop dynamics for $r(t)$ can be expressed by the following expression

$$M^*\dot{r} = -\frac{1}{2}\dot{M}^*r + Y_2\theta_2 + Y_1\tilde{\theta}_1 + \hat{J}U \quad (19)$$

where the definition of (15) has been utilized, $\tilde{\theta}_1 \triangleq \theta_1 - \hat{\theta}_1 \in \mathbb{R}^{p_3}$ represents the parameter estimation error with $\hat{\theta}_1(t)$ denoting the vector of parameter estimates, the terms $\frac{1}{2}\dot{M}^*(\eta)r(t)$ and $\hat{J}(\eta, \hat{\theta}_1)U(t)$ have been added/subtracted to the right hand side of (19), $Y_2\theta_2 \in \mathbb{R}^3$ captures the uncertainties associated with the elements of $M^*(\eta)$ and D^* and is explicitly defined as

$$Y_2\theta_2 = \frac{1}{2}\dot{M}^*r + M^*(\alpha\dot{e} - \ddot{\eta}_d) - D^* \quad (20)$$

where $Y_2(\cdot) \in \mathbb{R}^{3\times p_2}$ is a known, measurable regression vector and $\theta_2 \in \mathbb{R}^{p_2}$ is a vector of p_2 unknown constants. $\hat{J}(\eta, \hat{\theta}_1) \in \mathbb{R}^{3\times3}$ represents a dynamic estimate of the input related matrix $J(\eta, \theta_1)$ and is constructed from the following on-line update law

$$\dot{\hat{\theta}}_1 = proj\{y\} \quad (21)$$

where the auxiliary term $y(t) \in \mathbb{R}^{p_1}$ is defined as

$$y = \Gamma_1 Y_1^T r \quad (22)$$

with $\Gamma_1 \in \mathbb{R}^{p_1\times1}$ denoting a constant positive diagonal gain matrix. The function $proj\{y\}$ introduced in (21) is defined in the following manner

$$proj\{y_i\} \triangleq \begin{cases} y_i & \text{if } \hat{\theta}_{1i} > \underline{\theta}_{1i} \\ y_i & \text{if } \hat{\theta}_{1i} = \underline{\theta}_{1i} \quad \text{and } y_i > 0 \\ 0 & \text{if } \hat{\theta}_{1i} = \underline{\theta}_{1i} \quad \text{and } y_i < 0 \\ 0 & \text{if } \hat{\theta}_{1i} = \overline{\theta}_{1i} \quad \text{and } y_i > 0 \\ y_i & \text{if } \hat{\theta}_{1i} = \overline{\theta}_{1i} \quad \text{and } y_i \leq 0 \\ y_i & \text{if } \hat{\theta}_{1i} < \overline{\theta}_{1i} \end{cases} \quad (23)$$

$$\underline{\theta}_{1i} \leq \hat{\theta}_{1i}(0) \leq \overline{\theta}_{1i} \quad (24)$$

where $y_i(t) \in \mathbb{R}$ denotes the ith component of $y(t)$, and $\hat{\theta}_{1i}(t) \in \mathbb{R}$ denotes the ith component of $\hat{\theta}_1(t)$. The above projection algorithm is utilized to guarantee that $\hat{J}(\eta, \hat{\theta}_1)$ is invertible by ensuring the parameter estimates remain bounded in the sense that $\underline{\theta}_{1i} \leq \hat{\theta}_{1i}(t) \leq \overline{\theta}_{1i}$. For further details of the projection based update algorithm the reader is referred [16]).

Remark 4: At this point, it should be noted that the projection based update law of (23) is the source for the requirement of communication between tugs. Since placement of tugboats to ensure that $J(\eta, \theta_1)$ is diagonal is difficult to achieve, the regression vector $Y_1(U, \eta)$ of (15) is comprised of combinations of the tugboat thrusts thereby requiring the passing of their thrust magnitude among all the other tug members.

Based on (19) and the subsequent stability analysis the thrust input $U(t)$ is designed in the following manner

$$U = \hat{J}^{-1} \left(-Y_2\hat{\theta}_2 - Kr \right) \qquad (25)$$

where $K \in \mathbb{R}^{3\times3}$ denotes a positive constant diagonal gain matrix and $\hat{\theta}_2(t) \in \mathbb{R}^{p_2}$ denotes the parameter estimate for θ_2 and is calculated from the following dynamics

$$\dot{\hat{\theta}}_2 = \Gamma_2 Y_2^T r. \qquad (26)$$

where $\Gamma_2 \in \mathbb{R}^{p_2 \times p_2}$ represents a constant positive diagonal gain matrix. After substituting (25) into (19), the closed loop tracking error dynamics for $r(t)$ can be expressed as

$$M^*\dot{r} = -Kr - \frac{1}{2}\dot{M}^*r + Y_1\tilde{\theta}_1 + Y_2\tilde{\theta}_2 \qquad (27)$$

where $\tilde{\theta}_2(t) \triangleq \theta_2 - \hat{\theta}_2 \in \mathbb{R}^{p_2}$ denotes the parameter estimation error.

C. Stability Analysis

Theorem 1: Given the dynamic model of (13), the thrust control input $U(t)$ specified in (25) coupled with the parameter update laws of (21) and (26) ensure that the vessel tracking error signal $e(t)$ is driven asymptotically to zero in the sense that $\|e(t)\| \to 0$ as $t \to \infty$.

Proof: See Appendix I. ∎

V. CASE II: NO COMMUNICATION BETWEEN TUGBOATS

In order to eliminate the passing of information among tugboats which is required in the adaptive control design, a robust control strategy is developed for the position tracking objective that does not require the presence of a communication network. However in order to achieve this goal. the tugboats are required to be strategically placed around the hull in the manner of (6) such that the thrust configuration matrix B of (5) is upper triangular. If this tug placement is achieved and maintained, then a hierarchical approach is followed whereby "upper level" tugs treat thrust inputs from "lower level" tugs as disturbances (this of course motivates the utilization of a robust control structure). As in the adaptive control development, each tug is aware of its own location with respect to the vessel's center of mass, is preprogrammed with the vessel's desired maneuver trajectory $\eta_d(t)$, and has the ability to calculate the vessel's inertial position/velocity signal $\eta(t)$ and $\dot{\eta}(t)$, respectively.

A. Error System Development

To promote the control objective, the following position tracking error signal $z_1(t) \in \mathbb{R}^3$ is defined as

$$z_1 \triangleq R^T (\eta - \eta_d) \qquad (28)$$

where $R(\psi) \in \mathbb{R}^{3\times3}$ is the rotation matrix previously defined in (10), and the desired trajectory and its first three derivatives $\eta_d(t), \dot{\eta}_d(t), \ddot{\eta}_d(t), \dddot{\eta}_d(t) \in \mathbb{R}^3$ are known by all the tugboats and are bounded functions of time. In order to ease the design and analysis, the filtered error signal $z_2(t) \in \mathbb{R}^3$ is defined in the following manner

$$z_2 \triangleq R^T (\dot{\eta} - \dot{\eta}_d) + K_1 z_1 \qquad (29)$$

where $K_1 \in \mathbb{R}^{3\times3}$ denotes a positive constant diagonal gain matrix. From (28), the open-loop position tracking error dynamics for $z_1(t)$ can be formulated as

$$\dot{z}_1 = -K_1 z_1 - \dot{\psi}Sz_1 + z_2 \qquad (30)$$

where S is a skew-symmetric matrix previously defined in (12), and the expressions of (11) and (28) have been utilized. After taking the time derivative of (29) and multiplying both sides of the equation by $M(\nu)$, the following expression can be obtained for the open-loop filtered tracking tracking error dynamics

$$M\dot{z}_2 = M\dot{\nu} + M\frac{d}{dt}\left\{K_1 z_1 - R^T\dot{\eta}_d\right\} \qquad (31)$$

where the kinematic relationship of (8) has been employed. After adding and subtracting the terms $z_1(t)$, $\frac{1}{2}\dot{M}(\nu)z_2(t)$ to (31), and substituting in (9), the following open loop error system for $z_2(t)$ is obtained

$$M\dot{z}_2 = -\frac{1}{2}\dot{M}z_2 - z_1 + N + BU \qquad (32)$$

where the auxiliary term $N(\cdot) \in \mathbb{R}^3$ is defined in the following manner

$$N = \frac{1}{2}\dot{M}z_2 + z_1 - D + M\frac{d}{dt}\left\{K_1 z_1 - R^T\dot{\eta}_d\right\}. \qquad (33)$$

Based on (32) and the subsequent stability analysis, the thrust input vector $U(t)$ is designed as

$$U = -\bar{B}\left(K_2 z_2 + \hat{F}\right) \qquad (34)$$

where $K_2 \in \mathbb{R}^{3\times3}$ is a positive constant diagonal gain matrix, $\hat{F}(t) \in \mathbb{R}$ is a neural network based feedforward term which is specified in a manner to be bounded[1], and the diagonal matrix $\bar{B} \in \mathbb{R}^{3\times3}$ is explicitly given as

$$\bar{B} = \begin{bmatrix} 1/b_{11} & 0 & 0 \\ 0 & 1/b_{22} & 0 \\ 0 & 0 & 1/b_{33} \end{bmatrix} \qquad (35)$$

where $b_{11}, b_{22}, b_{33} \in \mathbb{R}$ are the diagonal elements of the thrust configuration matrix of (5). After substituting the thrust input designed in (34) into the open loop dynamics of (32), the following closed loop error system for $z_2(t)$ is developed

$$\begin{aligned} M\dot{z}_2 =\ & -K_2 z_2 - \frac{1}{2}\dot{M}z_2 - z_1 + \tilde{N}_1 + N_{1d} \\ & + \begin{bmatrix} 0 & b_{12}/b_{22} & b_{13}/b_{33} \\ 0 & 0 & b_{23}/b_{33} \\ 0 & 0 & 0 \end{bmatrix} K_2 z_2 \end{aligned} \qquad (36)$$

where b_{ij} represents the element from the ith row and j^{th} column of the matrix B, $\tilde{N}_1 \triangleq N_1 - N_{1d}$, and $N_1(\cdot) \in \mathbb{R}^3$ are defined as follows

$$N_1 = N - B\bar{B}\hat{F}. \qquad (37)$$

[1]For details of the structure of the bounded neural network feedforward term, see Appendix II.

In (36), the term $N_{1d}(\cdot) \in \mathbb{R}^3$ is defined as follows

$$N_{1d} = N_1|_{\eta=\eta_d, \dot{\eta}=\dot{\eta}_d} \quad (38)$$

$$= -D - M\frac{d}{dt}\left\{R^T\dot{\eta}_d\right\} - B\bar{B}\hat{F}. \quad (39)$$

Note that from Property 1, $N_{1d}(t)$ can be shown to be bounded if $\nu(t), \hat{F}(t), \eta_d(t), \dot{\eta}_d(t), \ddot{\eta}_d(t) \in \mathcal{L}_\infty$. Using the fact that the desired trajectory and its corresponding time derivatives are bounded, it can be shown that $\tilde{N}_1$ can be upper bounded in the following manner

$$\tilde{N}_1 \le \rho\left(\|z\|\right)\|z\| \quad (40)$$

where $z = \left[z_1^T, z_2^T\right]^T \in \mathbb{R}^6$ and $\rho(\cdot)$ is a globally invertible, positive, non-decreasing function.

Remark 5: The thrust control input defined in (34) requires knowledge of the diagonal elements of the thrust configuration matrix. If we expand (34), it can be observed that the first element of $U(t)$ is defined as $u_1 = (1/b_{11})\left(k_{21}z_{21} + \hat{f}_1\right)$, where k_{21} is the first diagonal element of K_2, $z_{21}(t)$ is the first element of $z_2(t)$, and $\hat{f}_1(t)$ is the first element of $\hat{F}(t)$. From the commutation strategy of (7a)-(7b), $u_1(t)$ is used solely to compute the thrust inputs for the tugboats $u_{1a}(t)$ and $u_{1b}(t)$. Under the assumption that each tug knows its own position along the vessel's hull (*i.e.* b_{11} will be known by the controller $u_1(t)$) and from the decomposition of the thrust configuration matrix presented in Section II, it is easy to observe that $u_1(t)$ can be calculated without knowing any other elements of the thrust configuration matrix and as a result does not require information of the other tugboats locations.

B. Stability Analysis

Theorem 2: For the vessel model of (8)-(9), the thrust control input of (34) ensures that the position tracking error is globally uniformly ultimately bounded (GUUB) in the sense that

$$\|z_1(t)\|, \|z_2(t)\| \le \sqrt{\beta_0 \exp(-\lambda_3 t) + \beta_1} \quad (41)$$

where $\beta_0, \beta_1, \lambda_3 \in \mathbb{R}^+$ denote constants.

Proof: In order to illustrate the result of (41), the following non-negative scalar function $V_2(t) \in \mathbb{R}$ is defined as

$$V_2 = \frac{1}{2}z_1^T z_1 + \frac{1}{2}z_2^T M(\nu)z_2 \quad (42)$$

After taking the time derivative of (42), substituting from (30) and (36), the following expression can be obtained for the time derivative of $V_2(t)$

$$\begin{aligned}
\dot{V}_2 = &-z_1^T K_1 z_1 - z_2^T K_2 z_2 + z_2^T \tilde{N}_1 + z_2^T N_{1d} \\
&+ k_{22}\left(\frac{b_{12}}{b_{22}}\right)z_{21}z_{22} + k_{23}\left(\frac{b_{13}}{b_{33}}\right)z_{21}z_{23} \\
&+ k_{23}\left(\frac{b_{23}}{b_{33}}\right)z_{22}z_{23}
\end{aligned} \quad (43)$$

where z_{2i} is the ith element of z_2, k_{2i} is the ith diagonal element of K_2. The expression for $\dot{V}_2(t)$ in (43) can be upper

bounded in the following manner

$$\begin{aligned}
\dot{V}_2 \le &-z_1^T K_1 z_1 - z_2^T K_n z_2 + \rho(\|z\|)\|z\|^2 \\
&- \left(k_{n1}|z_{21}|^2 - k_{22}\left(\frac{b_{12}}{b_{22}}\right)|z_{21}||z_{22}|\right) \\
&- \left(k_{n1}|z_{21}|^2 - k_{23}\left(\frac{b_{13}}{b_{33}}\right)|z_{21}||z_{23}|\right) \\
&- \left(k_{n2}|z_{22}|^2 - k_{23}\left(\frac{b_{23}}{b_{33}}\right)|z_{22}||z_{23}|\right) \\
&+ \left(\|N_{1d}\|\|z_2\| - \lambda_{min}\{K_n\}\|z_2\|^2\right)
\end{aligned} \quad (44)$$

where $K_n \in \mathbb{R}^{3\times3}$ is a constant positive diagonal gain matrix which is selected such that $K_2 = 5K_n$, k_{ni} is the ith diagonal element of K_n, and λ_{min} represents the minimum eigenvalue. After completing the squares on the bracketed terms of (44), the following upper bound for the expression in (44) can be developed

$$\begin{aligned}
\dot{V}_2 \le &-z_1^T K_1 z_1 - k_{n1}|z_{21}|^2 - k_{n2}|z_{22}|^2 - k_{n3}|z_{23}|^2 \\
&+ \frac{k_{22}^2}{k_{n1}}\left(\frac{b_{12}}{b_{22}}\right)^2|z_{22}|^2 + \frac{k_{23}^2}{k_{n1}}\left(\frac{b_{13}}{b_{33}}\right)^2|z_{23}|^2 \\
&+ \frac{k_{23}^2}{k_{n2}}\left(\frac{b_{23}}{b_{33}}\right)^2|z_{23}|^2 + \rho(\|z\|)\|z\|^2 + \epsilon_0
\end{aligned} \quad (45)$$

where $\epsilon_0 \ge \frac{1}{\lambda_{min}\{K_n\}}\|N_{1d}\|^2 \in \mathbb{R}^+$. The expression of (45) can be further upper bounded in the following manner

$$\begin{aligned}
\dot{V}_2 \le &-\lambda_{min}\{K_1\}\|z_1\| - k_{n1}|z_{21}|^2 - k_p|z_{22}|^2 \\
&- k_q|z_{23}|^2 + \rho(\|z\|)\|z\|^2 + \epsilon_0
\end{aligned} \quad (46)$$

where k_p, k_q are positive scalar constants which are defined as follows

$$\begin{aligned}
k_p &= k_{n2} - (k_{22}^2/k_{n1})(b_{12}/b_{22})^2, \\
k_q &= k_m - (k_{23}^2/k_{n1})(b_{13}/b_{33})^2, \\
k_m &= k_{n3} - (k_{23}^2/k_{n2})(b_{23}/b_{33})^2.
\end{aligned} \quad (47)$$

The control gains are selected as follows to ensure that the variables of (47) are positive

$$\begin{aligned}
&k_{n1} > 25\left(\frac{b_{12}}{b_{22}}\right)^2 k_{n2}, \qquad k_{n2} > 25\left(\frac{b_{23}}{b_{33}}\right)^2 k_{n3} \\
&k_{n1} > 25\left(\frac{b_{13}}{b_{33}}\right)^2\left(\frac{k_{n3}^2}{k_m}\right),
\end{aligned} \quad (48)$$

The combination of the four terms of (46) allows the expression for $\dot{V}_2(t)$ to be upper bounded as follows

$$\dot{V}_2 \le -\left[\lambda_1 - \rho(\|z\|)\right]\|z\|^2 + \epsilon_0 \quad (49)$$

where $z(t)$ was previously defined in (40) and $\lambda_1 \in \mathbb{R}^+$ represents a constant defined as follows

$$\lambda_1 = \min\{\lambda_{min}\{K_1\}, \min\{k_{n1}, k_p, k_q\}\}. \quad (50)$$

Provided that the gain values are selected to ensure that $\lambda_1 > \rho(\|z\|)$, then the time derivative of $V_2(t)$ can be upper bounded as

$$\dot{V}_2 \le -\lambda_2\|z\|^2 + \epsilon_0 \quad (51)$$

where $\lambda_2 = \lambda_1 - \rho(\|z\|) \in \mathbb{R}^+$. From (42), (51) can be upper bounded as follows

$$\dot{V}_2 \leq -\lambda_3 V_2 + \epsilon_0 \tag{52}$$

where $\lambda_3 \in \mathbb{R}^+$ is defined as follows

$$\lambda_3 = \frac{2\lambda_2}{\max\{1, \lambda_{max}\{M(\nu)\}\}}. \tag{53}$$

The differential inequality of (52) can now be utilized to formulate an upper bound on $V_2(t)$ as follows

$$V_2(t) \leq V_2(0)\exp(-\lambda_3 t) + \frac{\epsilon_0}{\lambda_3}\left[1 - \exp(-\lambda_3 t)\right]. \tag{54}$$

Based on (42), (52), and the definition of $z(t)$, (54) can be rewritten in the following form

$$\|z(t)\|^2 \leq \beta_0 \exp(-\lambda_3 t) + \beta_1 \tag{55}$$

where $\beta_0, \beta_1 \in \mathbb{R}^+$ are defined as

$$\beta_0 = \frac{\max\{1, \lambda_{max}\{M(\nu)\}\}}{\min\{1, \lambda_{min}\{M(\nu)\}\}}\|z(0)\|^2$$

$$\beta_1 = \frac{2\epsilon_0}{\min\{1, \lambda_{min}\{M(\nu)\}\}\lambda_3}.$$

From (55) and the definition of $z(t)$, the result given in (41) can be observed. Since $z_1(t) \in \mathcal{L}_\infty$, it is known that $(\eta - \eta_d) \in \mathcal{L}_\infty$, and hence, the position tracking result is achieved. Given (41), (30) and (36) can be utilized to prove that $\dot{z}_1(t), \dot{z}_2(t) \in \mathcal{L}_\infty$. After using the fact that $z_2(t), \hat{F}(t) \in \mathcal{L}_\infty$, the expression of (34) can be employed to prove that $U(t) \in \mathcal{L}_\infty$. $\blacksquare$

VI. SIMULATION RESULTS

In order to verify operation and to evaluate performance of the both the adaptive and robust control strategies, simulations were performed within the MATLAB SIMULINK environment utilizing the model parameter values of Cybership II [3] for the disabled vessel dynamics. In order to account for the unidirectional thrust input constraint, six tugboats were positioned at the following locations with respect to the vessel's center of mass

$$\begin{aligned} L_{1a} &= (-0.6275, 0), & L_{1b} &= (0.6275, 0), \\ L_{2a} &= (-0.25, -0.145), & L_{2b} &= (0.25, 0.145), & (m) \\ L_{3a} &= (-0.25, 0.145), & L_{3b} &= (0.25, -0.145). \end{aligned}$$

In addition, the incident angle of each tugboat with respect to the hull was selected according to (6) in order to ensure that the resulting configuration matrix is upper triangular and are given as follows

$$\begin{aligned} \alpha_{1a} &= 0, & \alpha_{1b} &= \pi, & \alpha_{2a} &= \pi/4, \\ \alpha_{2b} &= \alpha_{2a} + \pi, & \alpha_{3a} &= 7\pi/4, & \alpha_{3b} &= \alpha_{3a} - \pi. \end{aligned} \quad (rad)$$

The desired position trajectory for both control strategies was selected as follows

$$\eta_d = [0, 0, (\pi/2)(1 - \exp(0.1t))]^T$$

with the initial position/orientation of the vessel set to $\eta(0) = [1.0, 1.0, \pi/8]^T$. The control gain values were selected according to the conditions set forth in the stability

analysis through a trial and error procedure that produced the best overall position tracking performance. For the adaptive control strategy of Section IV, the control gains were selected in the following manner

$$\begin{aligned} K &= diag\{20.0, 20.0, 20.0\}, & \alpha &= diag\{1.0, 1.0, 1.0\}, \\ \Gamma_1 &= 20.0 \cdot \mathrm{I}_{12}, & \Gamma_2 &= 2.0 \cdot \mathrm{I}_7 \end{aligned}$$

where $\mathrm{I}_{12} \in \mathbb{R}^{12 \times 12}$ and $\mathrm{I}_7 \in \mathbb{R}^{7 \times 7}$ represent identity matrices. The initial estimates $(\hat{\theta}_1(t) \in \mathbb{R}^{13}, \hat{\theta}_2(t) \in \mathbb{R}^7)$ were initialized as follows

$$\hat{\theta}_1(0) = \underline{\theta}_1 \quad \hat{\theta}_2(0) = 0.0$$

where $\underline{\theta}_1$ was assumed to be 25% of the true value θ_1. For the robust control strategy, the control gain values were selected as follows

$$K_1 = diag\{1.0, 1.0, 1.0\}, \quad K_2 = diag\{500.0, 100.0, 2.0\}.$$

Figures 2 - 4 and Figures 7 - 9 illustrate the position tracking results obtained from adaptive and robust control strategies, respectively. As seen from the position tracking error Figures 2 and 7, both controllers are able to maneuver the vessel along the desired vessel tracking. It is also interesting to observe the unidirectional tugboat thrust inputs of Figures 4 and 9. As seen in these figures even when the tracking error signal is close to zero, a small amount of thrust is maintained to keep the tugboat in contact with the vessel hull when not in use (this is the result of the small constant γ_0 present in the commutation strategy of (7a) and (7b). For both simulations the constant was selected as $\gamma_0 = \sqrt{5}$).

VII. CONCLUSION

In this paper, two control strategies have been presented for the positioning of an unactuated vessel by multiple, autonomous tugboats. Specifically, an adaptive position control strategy was developed to target the problem where the location of the tugboats about the vessel hull is not known. As a result of this uncertainty, communication between members of the tugboat group is required to adequately compensate for the unknown thrust matrix. To offer attractiveness for recovery of disabled vessels within hostile environments, a second robust control strategy is developed that does not require the passing of data between the tugboats. To facilitate this lack of communication/coordination, the tugboat locations about the vessel are selected *a priori* in a manner such that the resulting thrust configuration matrix is upper triangular. This structure allows for an hierarchical control design approach where "upper level" tugboats treat thrust inputs from "lower level" tugs as disturbances. Both control approaches have been verified via simulation. There is much work to be considered for future work. One of the major assumptions of this work is that the tugboat locations and their incident angles to the hull remain fixed throughout the maneuver; therefore, subsequent work will include tugboat dynamics within the scope of the control design. Also, the control design must also account for the possibility of the tugboat slipping along the hull. As a result, future control strategies should include the constraint that lateral forces

along the hull from the tug remain below the contact static friction coefficient [17]. In addition, work is under way for the validation of both control strategies via experiments.

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APPENDIX I
PROOF OF THEOREM I

Proof: In order to illustrate the position tracking result, the following non-negative scalar function $V_1(t) \in \mathbb{R}$ is defined as follows

$$V_1 = \frac{1}{2}r^T M^* r + \frac{1}{2}\tilde{\theta}_1^T \Gamma_1^{-1}\tilde{\theta}_1 + \frac{1}{2}\tilde{\theta}_2^T \Gamma_2^{-1}\tilde{\theta}_2. \qquad (56)$$

After taking the time derivative of (56), substituting from (27), the following expression for the time derivative of $V_1(t)$ can be obtained

$$\dot{V}_1 = -r^T K r + \tilde{\theta}_1^T \left[Y_1^T r - \Gamma_1^{-1}\dot{\hat{\theta}} \right] \qquad (57)$$
$$+\tilde{\theta}_2^T \left[Y_2^T r - \Gamma_2^{-1}\dot{\hat{\theta}}_2 \right]$$

After substituting the projection based update laws of (21) and (26) into (57), the dynamics for $V(t)$ can be upper bounded in the following manner

$$\dot{V}_1 \leq -\lambda_{min}\{K\}\|r\| \qquad (58)$$

where $\lambda_{min}\{K\}$ represents the minimum eigenvalue of the gain matrix K.

From the above expressions of (56) and (58), the signals $r(t), \tilde{\theta}_1(t), \tilde{\theta}_2(t)$ can be observed to be bounded. Since $r(t) \in \mathcal{L}_\infty$, the definition of $r(t)$ of (18) can then be utilized to show that position/velocity tracking error signals are also bounded $(e(t), \dot{e}(t) \in \mathcal{L}_\infty)$. Since the desired trajectory was selected such that $\eta_d(t), \dot{\eta}_d(t) \in \mathcal{L}_\infty$, $\eta(t), \dot{\eta}(t) \in \mathcal{L}_\infty$. Also since $\tilde{\theta}_1(t), \tilde{\theta}_2(t) \in \mathcal{L}_\infty$, the parameter estimates $\hat{\theta}_1(t)$ and $\hat{\theta}_2(t)$ are bounded signals (note that $\hat{\theta}_1(t)$ is bounded independent of the stability analysis since the projection based update algorithm of (23) was utilized). Since $r(t), \dot{e}(t) \in \mathcal{L}_\infty$, Property 1 and (20) is utilized to show that $Y_2(t) \in \mathcal{L}_\infty$; hence $U(t)$ can be observed to be a bounded signal from (25). At this point, (21), (22), (26), the projection based update algorithm of (23), and previous arguments can be utilized to show that to show that $\dot{\hat{\theta}}_1, \dot{\hat{\theta}}_2 \in \mathcal{L}_\infty$. Since $e(t), \dot{e}(t) \in \mathcal{L}_2 \cap \mathcal{L}_\infty$, Barbalat's Lemma [18] can be employed to illustrate the position tracking result in the sense that $e(t) \to 0$ as $t \to \infty$. ∎

APPENDIX II
NEURAL NETWORK FEEDFORWARD

The neural network feedforward component is computed using a two layer network using 'n' neurons. This feedforward term must always be bounded (i.e. $\hat{f}(t) \in \mathcal{L}_\infty$), to this end the neural network component is defined as follows [14]

$$\hat{f} = \hat{W}^T \bar{\sigma}\left(\hat{V}^T x\right). \qquad (59)$$

where $\hat{W}(t) \in \mathbb{R}^{n \times 3}$ and $\hat{V}(t) \in \mathbb{R}^{13 \times n}$ are estimated weight matrices, and $x(t) \in \mathbb{R}^{13}$ is the input vector to the neural network which is selected as $x = \begin{bmatrix} 1, & \eta_d^T, & \dot{\eta}_d^T, & \ddot{\eta}_d^T, & \dddot{\eta}_d^T \end{bmatrix}^T$ where $\eta_d(t), \dot{\eta}_d(t), \ddot{\eta}_d(t), \dddot{\eta}_d(t)$ represent the desired trajectory and its first three derivatives. The vector activation function $\bar{\sigma}(\cdot) \in \mathbb{R}^n \mapsto \mathbb{R}^n$ is defined as follows $\bar{\sigma}(\omega) = [\sigma(\omega_1), \sigma(\omega_2), \cdots, \sigma(\omega_n)]^T \in \mathbb{R}^n$ where $\omega = [\omega_1, \omega_2, \cdots, \omega_n]^T \in \mathbb{R}^n$ and $\sigma(s) : \mathbb{R} \mapsto \mathbb{R}$ is the sigmoid activation function defined as

$$\sigma(s) = \frac{1}{1 + \exp(-s)} \ . \qquad (60)$$

The gradient of the vector activation function, denoted by $\bar{\sigma}'(\cdot) \in \mathbb{R}^{n \times n}$ can be expressed in closed form as follows, [19]

$$\bar{\sigma}(\omega)' = diag\{\bar{\sigma}(\omega)\} [I - diag\{\bar{\sigma}(\omega)\}]. \qquad (61)$$

To ensure that the weights generated from the weight update laws are bounded, the update laws were defined as follows [14]

$$\dot{\hat{W}} = -\alpha_w \hat{W} + \gamma_1 \bar{\sigma}\left(\hat{V}^T x\right) \, \text{sat}\,(e_2 + \zeta)^T \qquad (62)$$

$$\dot{\hat{V}} = -\alpha_v \hat{V} + \gamma_2 x \left[\bar{\sigma}'\left(\hat{V}^T x\right) \hat{W} \text{sat}\,(e_2 + \zeta)\right]^T \qquad (63)$$

where $\alpha_v, \alpha_w \in \mathbb{R}^+$ are small constants, $\gamma_1, \gamma_2 \in \mathbb{R}^+$ are control gains which effect the learning speed, the function $sat(\xi) : \mathbb{R}^n \mapsto \mathbb{R}^n$ is a saturation function, and the auxiliary signal $\zeta(t) \in \mathbb{R}^3$ is a surrogate (i.e. a dirty derivative operation) for the signal $\dot{e}_2(t)$ which is defined as follows

$$\zeta = \frac{1}{\varepsilon}(e_2 - \eta) \qquad (64)$$

where $\varepsilon \in \mathbb{R}^+$ is a small constant, and the signal $\eta(t) \in \mathbb{R}^3$ is updated according to the following expression

$$\dot{\eta} = \frac{1}{\varepsilon}(e_2 - \eta). \qquad (65)$$

From equations (59)-(65) and the fact that the input vector to the neural network is bounded, it is easy to show that the weight matrices $\hat{W}(t)$ and $\hat{V}(t)$ are bounded, and hence, the output from the neural network, $\hat{f}(t)$, is bounded.

APPENDIX III
SIMULATION FIGURES

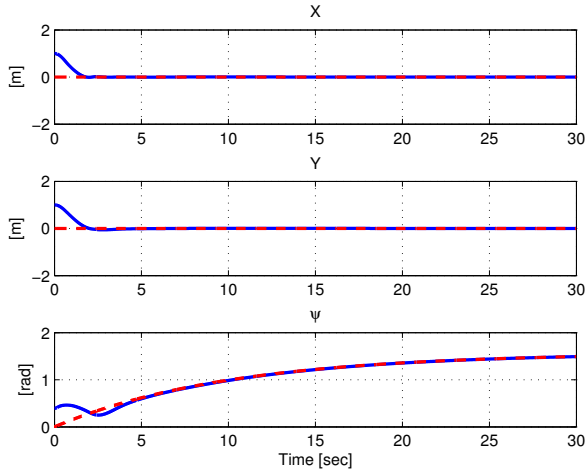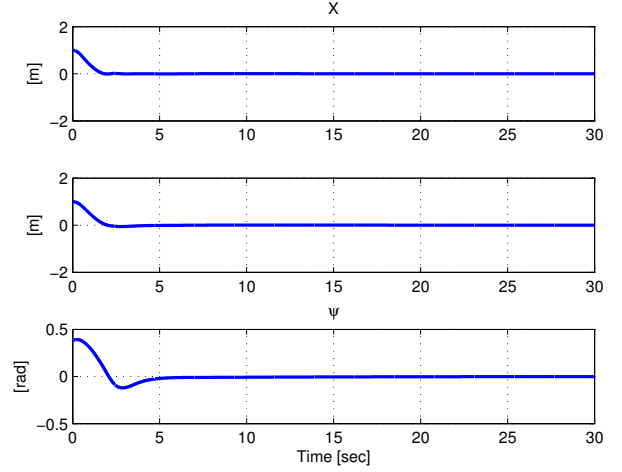Fig. 3: Tracking error for the case with full communication.

Fig. 4: Control input for the case with full communication, u_{ia} (solid line), u_{ib} (dashed line).

Fig. 2: Actual (solid line) and desired (dashed line) trajectories for the case with full communication.

Fig. 5: Dynamic parameter estimates $\hat{\theta}_1(t)$, for the case with full communication.

Fig. 6: Parameter estimates $\hat{\theta}_2(t)$, for the case with full communication.

Fig. 7: Actual (solid line) and desired (dashed line) trajectories for the case with no communication.

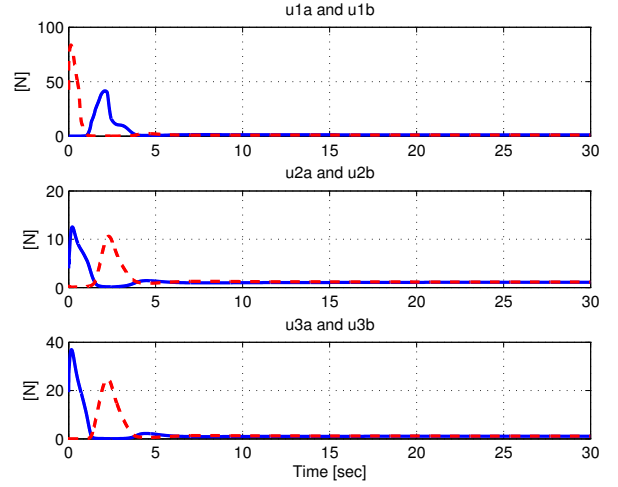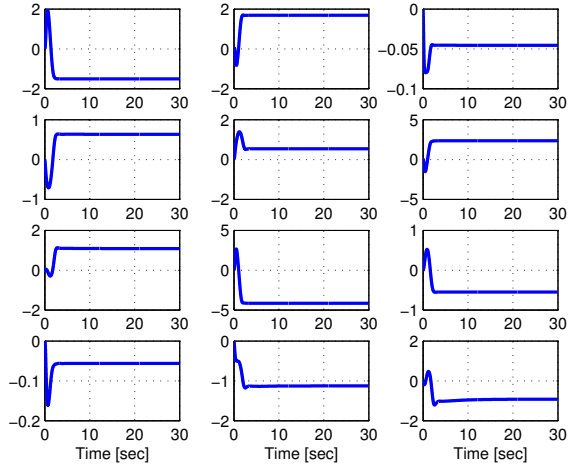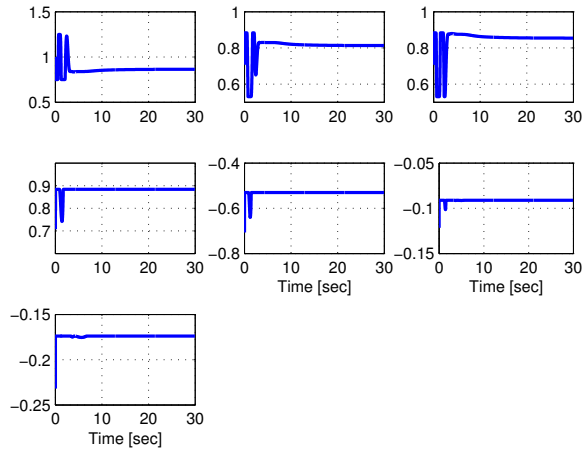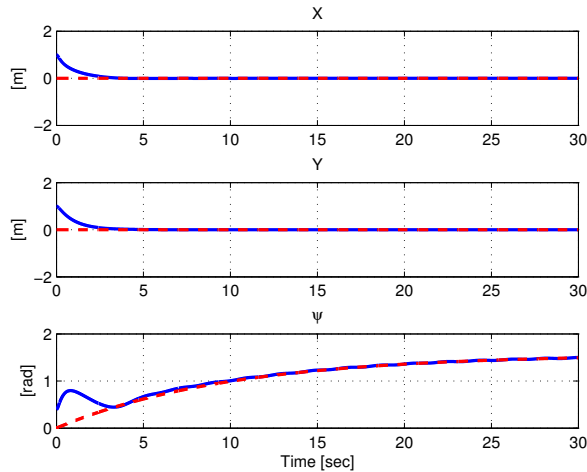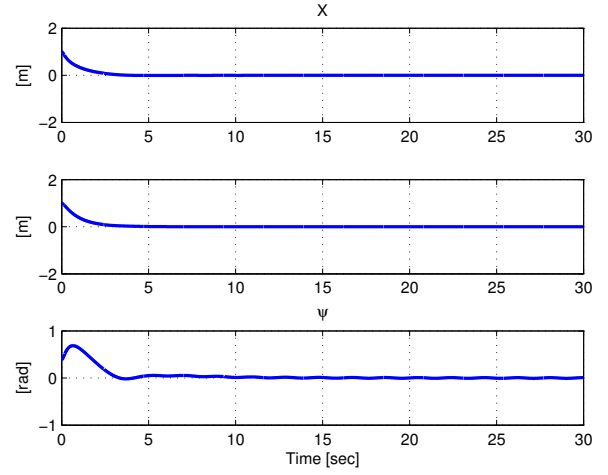
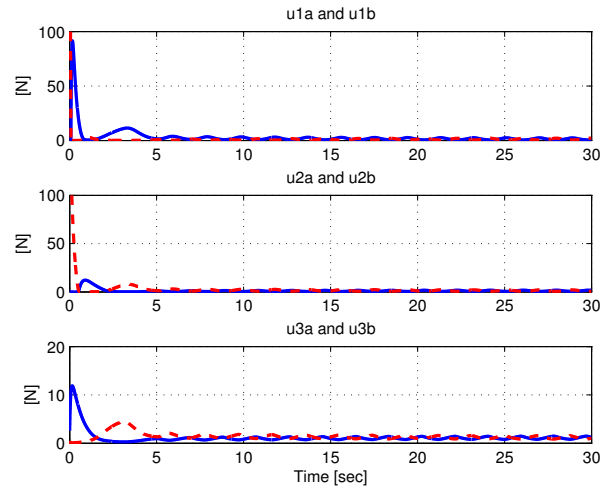Fig. 8: Tracking error for the case with no communication.

Fig. 9: Control input for the case with no communication, u_{ia} (solid line), u_{ib} (dashed line).